The Airborne Gyroscope

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Abstract

Gyroscopic motion involves two phenomena know as precession and nutation. Both were observed using a spherical gyroscope mounted on an air bearing so that the frictional forces were minimised. From these observations a relationship for precession was observed. The moment of inertia for the sphere was also calculated experimentally; we then compared this to the result from theory and found agreement.

1. Introduction

If the frictional forces are disregarded, a gyroscope will continue precessing and nutating forever, much like a pendulum, although instead of just gravity, the gyroscope relies on spin and gravity to maintain it's motion.

Gyroscopic motion governs how most things in the universe move, examples stem from how a Frisbee flies, to keeping the Mir space station at a certain orientation to the sun. A specific use of gyroscopic motion is the gyro compasses, where a gyroscope is used to keep the compass operational.

2. Theory

2.1. Precession

A simple example of a gyroscope is a sphere with a stalk attached, as shown in Figure 1.

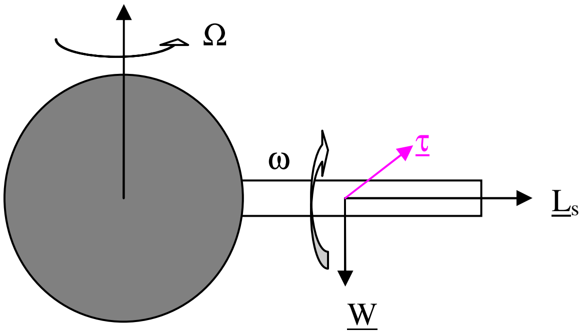


Figure 1: A simple spherical gyroscope

If the system is set spinning about the axis of the stalk (the x axis) the gyroscope will precess about the vertical axis of the sphere.

From the expression for the rate of precession; $\left|\frac{\delta \underline{L}_S}{\delta t}\right| = |\underline{\tau}|$ and by substituting in $\left|\frac{\delta \underline{L}_S}{\delta t}\right| = \Omega L_S$ and $|\underline{\tau}| = l|\underline{W}|$;

$$\Omega|\underline{L}_S| = l|\underline{W}|$$

The substitution $|\underline{L}_s| = I_0\omega_s$ gives

$$\Omega = \frac{|\underline{W}|l}{I_0\omega_S} \qquad \text{[Equation 1]}$$

It is important to remember that the centre of mass of the system is not located at the centre of mass of the sphere. This can be resolved by using the parallel axis theorem.

2.2. Nutation

If whilst precessing, the stalk is displaced in the vertical direction it will nutate. Nutation is best described as a nodding motion.

By plotting the angular velocity of the precessing stalk, $\dot{\phi}$, against the angular velocity of the spinning gyroscope, ω, the gradient of the linear relationship equals $\frac{I}{I_{O_x}}$.

$$a = \frac{I}{I_{O_x}} \qquad \text{[Equation 2]}$$

3. Method

3.1. Precession

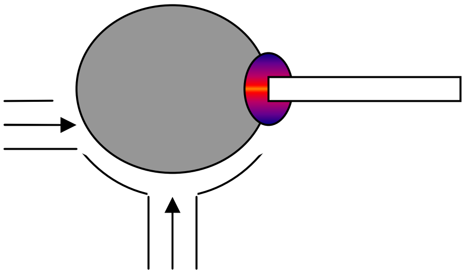


Figure 2: The spherical gyroscope mounted on an air bearing

To observe precession, the air bearing performs two jobs. Firstly a vertical air jet pushes the gyroscope upwards, this minimises the friction between the gyroscope and the bearing. Secondly, a horizontal air jet exerts a horizontal torque on the gyroscope, this makes the gyroscope spin. (see Figure 2)

Once the gyroscope is spinning, it is released so that it is free to precess. The precession frequency is then measured using a stopwatch and the frequency of rotation about the stalk is measured using a stroboscope.

3.2. Nutation

Using a similar set up to the precession the gyroscope is set spinning again. This time, in addition to precessing, the gyroscope is allowed to nutate. In addition to the frequency of rotation about the axis of the stalk, the frequency of nutation is measured using a second stroboscope.

4. Results

4.1.Errors

There are two types of errors;

- **Systematic errors**; which are flaws in the experiment. The systematic errors of this experiment include

  1. The friction on the gyroscope from the air bearing will slows the gyroscope down.

  2. The reading obtained could be a factor of the actual result. The actual result could be twice the frequency obtained, but the disk would still appear not to spin, as it will rotate twice per flash.

  The way to eliminate these systematic errors would be to use more accurate equipment and an environment that will minimise external forces even more so than the air bearing.

- **Random errors;** which are just imperfect readings due to the error of the equipment we can use, and the human error involved. These we can do something about. From our base measurements we evaluated the errors to be:

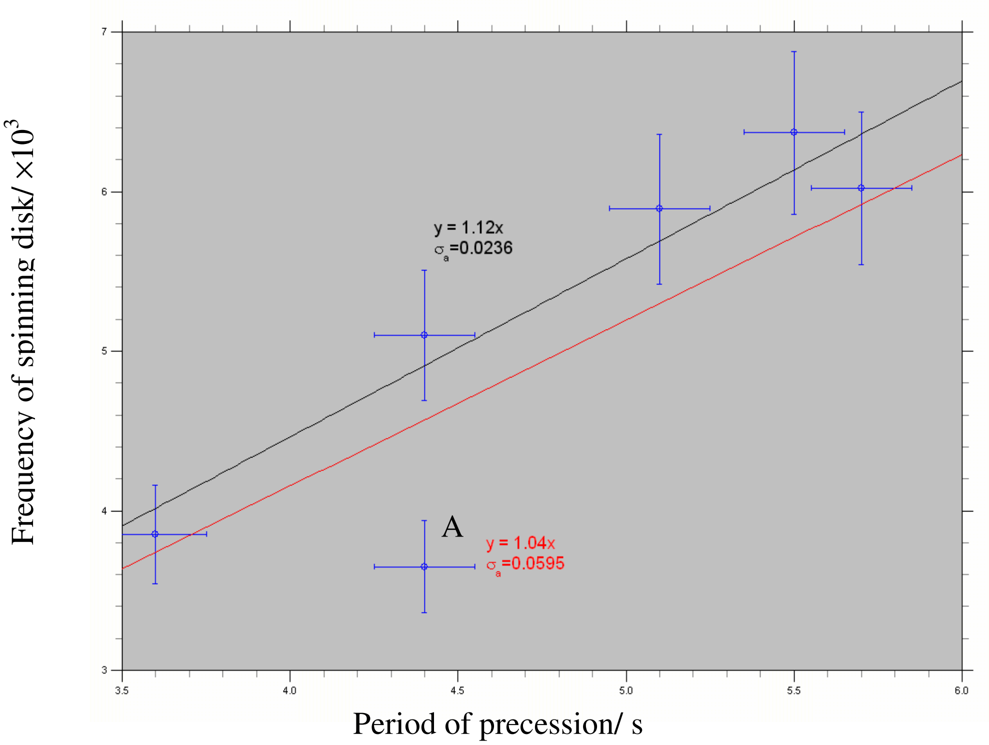
  - Error on Period to precess: ± 0.15 s
  - Error on period of strobe measuring disk spinning: $\pm 8\%$
  - Rod Length: 0.088 ± 0.0005 m
  - Mass of sphere: 0.5139 ± 0.00005 kg
  - Radius of sphere: 0.0251 ± 0.0001 m
  - Mass of rod: 0.011 ± 0.0005 kg
  - Nutational freq: $\pm 20\%$

The digital value of the period for the stroboscope was $\pm 0.0005s$. However this does not account for the large human error involved in judging weather the disk has stopped, therefore (as it is the experiment is highly unlikely to obtain what appears to be a completely stationary disk) we are required to modify this value significantly. Also because it is substantially harder to obtain accurate measurements when the disk is spinning very fast, the error will be modified into a percentage error of 8%.

So $\sigma_{\omega} = 8\%$

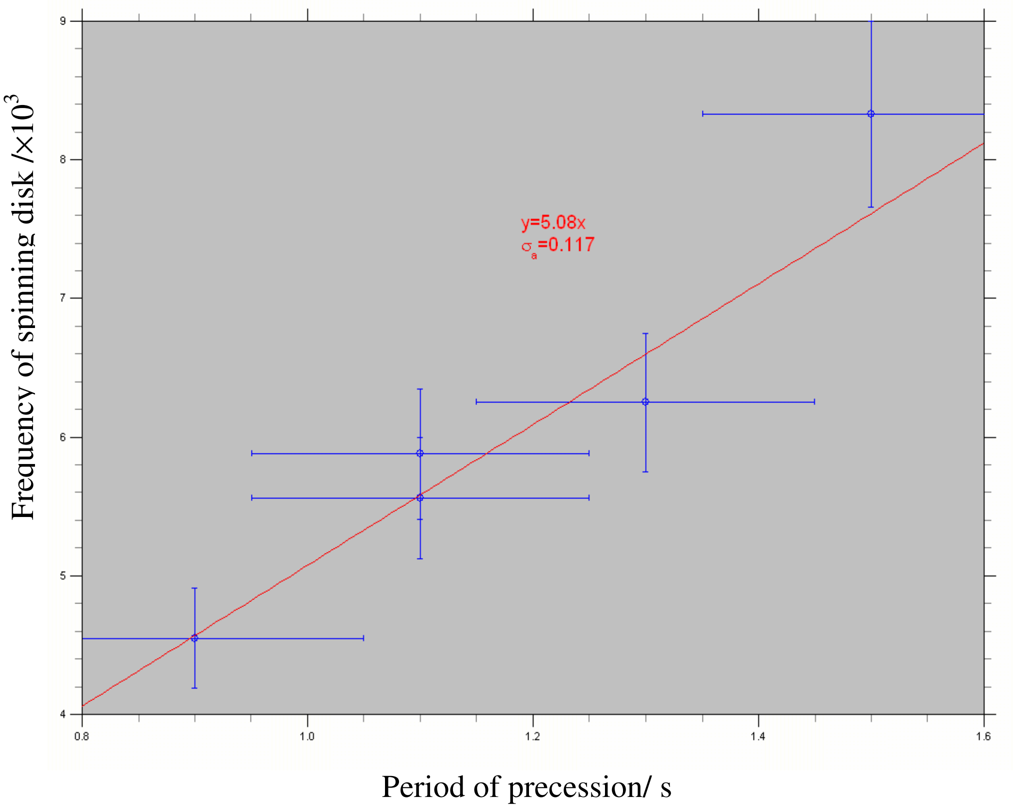
The angle from which the rod was released was also measured to the nearest 10 degrees. This did not affect the linear relationship of these two quantities. This value is superfluous so doesn't need perusing any further.
Originally the time for 10 periods was calculated to increase accuracy. However from the first results it became clear that by the time 10 periods have elapsed ω has degraded due to friction and so only one precession period at the time was measured.

4.2.Precession



Graph 1; Precession without biconical weight attached

[The red line includes point A, the black line ignores this point as it appears to be a rouge point]



Graph 2; Precession with biconical weight attached

Calculation of I_0 from graphs

By comparing the Equation $\Omega = \frac{|W|l}{I_0 \omega_S}$ to the equation of a straight line; $y = ax + b$ we find that we can plot a straight line by equating;

$$y = \Omega^{-1}$$
$$x = \omega$$
$$a = \frac{|W|l}{I_0}$$

For the case where the biconical weight was not attached (see Graph 1):

- From red line of best fit: $a = 1.04 \times 10^3$, $I_0 = (1.3 \pm 0.06) \times 10^{-4} m^2 kg$
- From black line of best fit: $a = 1.12 \times 10^3$, $I_0 = (1.2 \pm 0.02) \times 10^{-4} m^2 kg$

For the case where the biconical weight was attached (see graph 2):

$$a = 5.08 \times 10^3$$
$$I_0 = (3.17 \pm 0.12) \times 10^{-5} m^2 kg$$

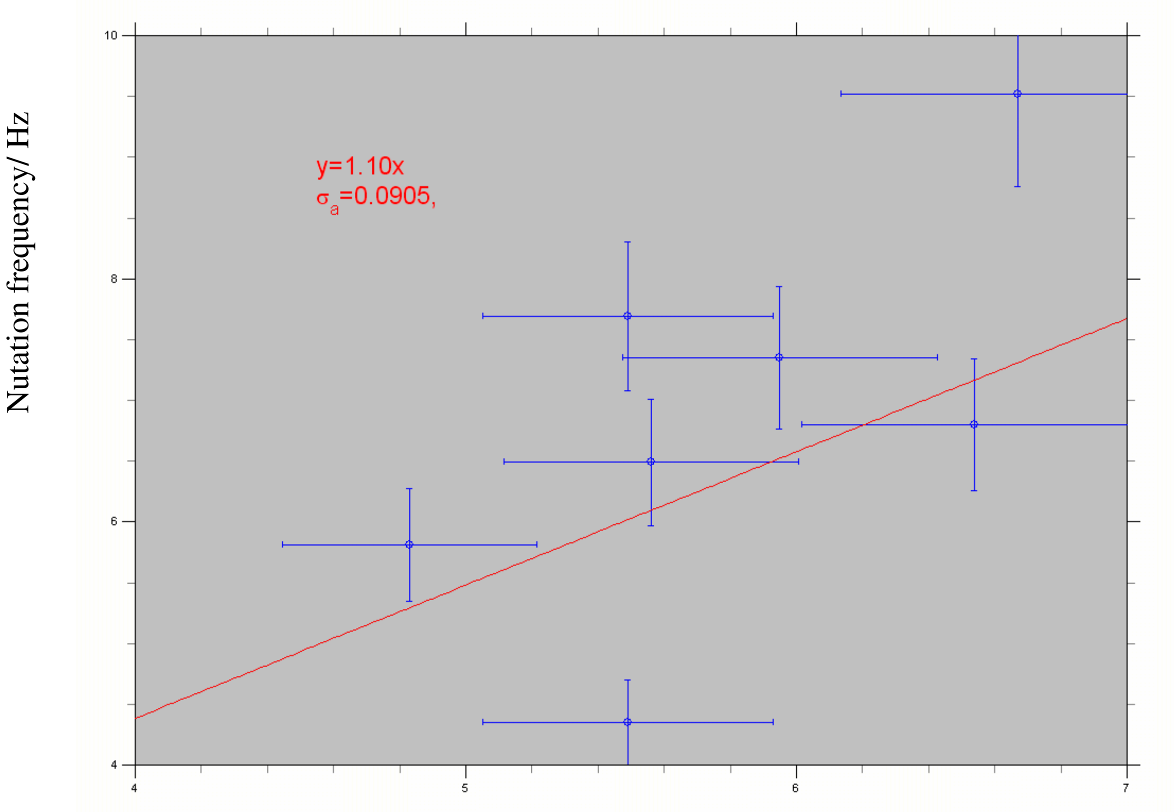
Theoretical Calculation of I_0 without biconical weight attached

$$I_o = \frac{3}{5} m r^2 = 5.81 \times 10^{-4} m^2 kg$$
$$\sigma_{I_o} = \left(\frac{3}{5} r \left(\sigma_m{}^2 r + 2 m \sigma_r{}^2 \right) \right)^{\frac{1}{2}} = 1.2 \times 10^{-4} m^2 kg$$
$$I_O = (5.81 \pm 1.2) \times 10^{-4} m^2 kg$$

Theoretical Calculation of I_0 with biconical weight attached

$$I_o = \frac{3}{5} m r^2 = 2.06 \times 10^{-4} m^2 kg$$
$$\sigma_{I_o} = \left(\frac{3}{5} r \left(\sigma_m{}^2 r + 2 m \sigma_r{}^2 \right) \right)^{\frac{1}{2}} = 0.16 \times 10^{-4} m^2 kg$$
$$I_O = (2.06 \pm 0.16) \times 10^{-4} m^2 kg$$

4.3. Nutation



Graph 3: Nutation frequency against spin frequency for a gyroscope without the biconical weight attached

Using the standard formulae for calculating the moment of inertia of a sphere and the error on said inertia[1];

$$I = \frac{3}{5}mr^2 = 19.4\times10^{-5}\,m^2kg \qquad \left[M = 0.5139 + 0.011 = 0.5249kg\right]$$

$$\sigma_I = \left(\frac{3}{5}r\left(\sigma_m{}^2 r + 2m\sigma_r{}^2\right)\right)^{\frac{1}{2}}$$

$$= \pm 1.2\times10^{-5}\,m^2kg$$

$$I = (1.98 \pm 0.12)\times10^{-4}\,m^2kg$$

Hence, by theoretical calculation; $\frac{I}{I_O} = \frac{1.98\times10^{-4}}{5.81\times10^{-4}} = 0.331$ $\left[I_O = (5.81 \pm 1.2)\times10^{-4}\,m^2kg\right]$

5. Discussion

In the precession experiment, the value moment of inertia given by the experiment is about a fifth of the value calculated theoretically. Also in the nutation experiment the value of $\frac{I}{I_O}$ is more that four times greater than the value calculated theoretically.

These differences are largely due to human error, the most evident case of human error being in the nutation experiment, when attempting to match the stroboscope to the frequency of nutation. Also another significant source of human error was in the harmonics of the nutational frequency, if the stroboscope were set to the second or third harmonic, then the value for that frequency would be two or three times larger than the true value.

6. Conclusion

6.1 Precession

From the experiment it was found that the moment of inertia without the biconical weight attached was $I_O = (5.81 \pm 1.2) \times 10^{-4} m^2 kg$ and $I_O = (5.81 \pm 1.2) \times 10^{-4} m^2 kg$ with the biconical weight attached.

A linear relationship was demonstrated, the graph of which passed through the origin, as required. This showed an agreement with Equation 1.

In the future, the errors could be reduced to show this relationship more clearly in a number of ways. Firstly we could repeat the same method, although this would require much time to return any significant improvement in the value of the moment of inertia.

The method could be modified such that the measurements are more accurate. For example a magnet could be attached to the end of the stalk, so that as the stalk precesses the changing magnetic flux could be measured so that the period of precession could be measured to a greater accuracy.

6.2 Nutation

From the experiment it was found that the value of $\frac{I}{I_O}$ was 1.10. This experiment we subject to greater human error than the precession experiment, the main reason for this was the use of the second stroboscope to measure the frequency of nutation. A method of improving this measurement would be to use video camera instead of strobes, from this the motion could be observe at a slower frame rate making it easier to make the various measurements.

7. References

1. Kleppner & Kolenkow, *An introduction to mechanics*, McGraw-Hill, Chapter 7

2. Young & Freedman, *University Physics*, Pearson Addison Wesley, P.386

3. Tinker & Lambourne, *Further Mathematics for the Physical Sciences*, John Wiley & Sons, 5.4.1

4. Jordan & Smith, *Mathematical Techniques*, Oxford, Chapters 2 & 3