

The gyroscope and its uses

Introduction

Every sixth form Physics student studies some form of dynamics, but usually their studies do not include rotating bodies. Rotating bodies are particularly interesting as they do not behave as may be expected from sixth form studies; if you were to push a rotating body, it is unlikely to move in the direction of your push. A gyroscope is basically a body that is rotating about an axis of symmetry. For scientific study they usually take the shape of a sphere or a disk

History

Despite gyroscopes being used as children's toys centuries before, it was not until 1743 when the first test of gyroscopes took place. This was as part of 'Serson's Speculum' this was basically an improved sextant. It was a device used in foggy conditions at sea to give alignment with the horizon. This idea of using gyroscopes as navigational aids has continued through to modern times, whilst there have been changes to the design of gyroscopes, the principle is fairly similar.

But gyroscopes are by no means limited to navigational aids. In fact since Serson's Speculum, gyroscopes have been manipulated for many uses, some of which are detailed later on.

Conservation of Angular Momentum

I spent much of last summer working in a greenhouse. Whilst I was there I noticed an interesting feature of the watering system. The sprinklers consisted of a vertical hose pipe connected to the sprinkler head, this head was made of two parts. There was a small plastic piece which was shaped so that as the water came out of the hose it rebounded off so that the plastic piece rotated and so spread the water out in all directions. The second part was a heavier brass ring which surrounded the plastic part.

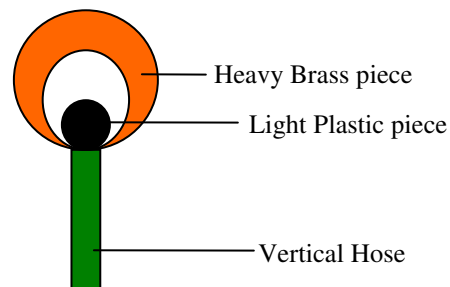


Diagram 1: Construction of sprinkler system

I believe this construction was actually designed to spread the water over the plants, but it was also a brilliant demonstration of angular momentum conservation.

The water came out the hose with only linear momentum; it was not rotating. It then rebounded off this plastic piece, which changed its linear momentum. From this picture we can assume that the plastic piece also underwent a change of linear momentum, but as it was fixed to the earth, such effects can be disregarded. This is the complete picture as far as linear momentum is

concerned.

Now if we think about the sprinkler head. The plastic piece is set rotating by the water. We now have the mass of the water rotating anticlockwise and the plastic piece rotating clockwise. Momentum is defined as the product of mass and the velocity. This is all fair for linear movements, but what about rotations? In this case we must use angular momentum. Much like it's linear counterpart, angular momentum must be conserved.

If we were to denote angular momentum by the symbol L ^[1], and for simplicity disregard the angular momentum of the water after it has collided with the sprinkler head, then for this situation:

$$\underline{L}_{\text{plastic piece}} + \underline{L}_{\text{brass piece}} = \underline{L}_{\text{water before contact with sprinkler}} = \underline{0}. \quad [\text{Equation 1}]$$

It is known that $\underline{L} = \left(\sum m_i r_i^2\right) \underline{\omega}$ where:

m_i = The mass of particle i in the system

r_i = The distance of the particle i from the rotation axis

$\underline{\omega}$ = The angular velocity vector

Hence by inserting this into Equation 1:

$$\left(\sum m_i r_i^2\right)_{\text{plastic piece}} \underline{\omega}_{\text{plastic piece}} + \left(\sum m_i r_i^2\right)_{\text{brass piece}} \underline{\omega}_{\text{brass piece}} = \underline{0} \quad [\text{Equation 2}]$$

The plastic piece has a much smaller mass and it's particles are much closer to the axis of rotation than for the brass piece. Hence using Equation 2 we find that the angular velocity of the plastic piece must be much larger than for the brass piece. i.e. We would expect the plastic piece to rotate in the opposite direction and to rotate much faster than the brass piece. This is what I observed.

Fleming's right hand rule

Much like it's linear counterpart, angular momentum is a vector which describes the movement of mass.

But a vector can be represented by an arrow, which can only point in one direction at any given point in time. So how is it possible to ascribe a single vector when the particles in the body all move with different instantaneous velocities?

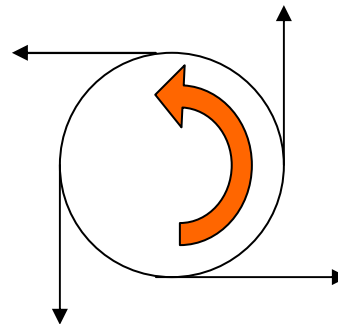


Diagram 2: Every particle of the spinning gyroscope is moving with a different velocity.

The answer is to use Fleming's right hand screwing rule. This allows us to use a single vector to describe whether the gyroscope is rotating clockwise or anticlockwise around the axis that the angular momentum vector lies upon.

If you curve the fingers on your right hand and point your thumb straight up to form the basis for Fleming's right hand rule. If your fingers curve in the direction that the gyroscope is spinning, then your thumb points in the direction of the angular momentum. Of course if you were to try this with your left hand, your angular momentum would be in the negative direction of the right handed answer, therefore it is important to remember to use your right hand.

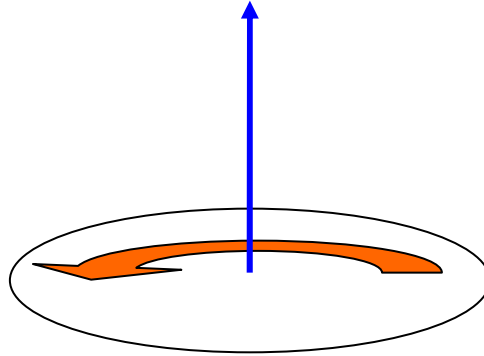


Diagram 3: Using Fleming's Right Hand Rule, our spinning gyroscope disk has an angular momentum which is perpendicular to the plane of the disk.

Using Fleming's right hand rule to find the resulting force when a moment is exerted on a gyroscope

So now we have our angular momentum vector, what use is it? If the body were just freely rotating under no forces (i.e. the body is out in space away from any other mass so that the body is weightless) then this is as far as we could go.

However, in reality, this is not the case. Let's take the simple case of a child's spinning top.

If the top were spun so that the angular momentum vector pointed vertically then the top would actually just continue to spin until so much kinetic energy was lost through friction that it fell over.

But, in reality, the axis of rotation is not vertical and so the weight of the top and the normal contact from the surface of the table, whilst parallel, are not collinear.

In this second scenario angular momentum is not yet conserved. As momentum is a quantity which must always be conserved, clearly something must be missing from this picture. To conserve angular momentum the body rotates around the vertical axis, this motion is known as precession. We can convince ourselves of precession using vectors.

If we take our child's spinning top and look down from above we find that the precession motion would be a circular rotation about the pivot at the bottom of the spinning top.

Using vector analysis we find that we can calculate the torque on a body using the dimensions of the body as; $\underline{\tau} = \underline{r} \times \underline{w}$, where:

$\underline{\tau}$: Torque on gyroscope

\underline{r} : Position vector from pivot to centre of mass

\underline{w} : The weight vector of the gyroscope

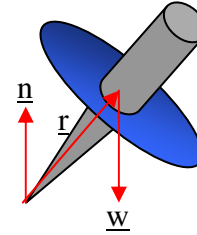


Diagram 4: The forces on a stationary gyroscope give the torque as pointing into the page.

If we now consider the change in the angular momentum we find that the torque and the change in angular momentum point in the same direction. As a result we find that the change in the angular momentum and the instantaneous angular momentum are perpendicular.

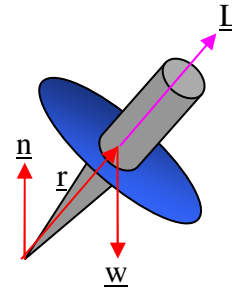


Diagram 5: For the spinning gyroscope we conclude that the torque and the change in the angular momentum both point in the same direction; into the page.

Finally we can show precession must occur by finding the angular momentum an instant later. We do this by adding the change in the angular momentum to the current angular momentum. As the change is always perpendicular to the current angular momentum we find that the gyroscope precesses through a circle. It is much easier to visualise this from above:

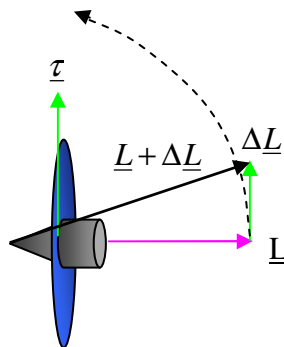


Diagram 6: Precession causes a gyroscope to rotate through a circle

It is this same resistance that the child's spinning top has to toppling over which is why it is easier to balance on a moving bicycle than a stationary one.

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However this is still not a complete picture of gyroscopes. If you consider the moment that the gyroscope is first released from its held position. The effects of precession take time and so in the first instant the gyroscope does drop towards the ground as we would expect if it were not spinning.

In a similar fashion to before, this dropping motion must be resisted to conserve angular momentum. The amount by which it drops depends upon the mass of the gyroscope and the rate at which it is spinning.

As a result what happens is that the gyroscope appears to rotate upwards. This motion sets about periodically and so the dropping movement repeats, which is then followed by the upwards movement and so on.

This movement is known as nutation, and when it is combined with precession we arrive at a final view of gyroscopic motion.

Uses of gyroscopes

Of course gyroscopes are used for more than just children's amusement. The fact that a gyroscope reacts to conserve momentum is manipulated to stabilise ships. A gyroscope is kept constantly spinning using air jets. When the ship tilts due to ocean waves, the angle by which the ship has been displaced can accurately be measured using magnetic induction. Using this information the ship's stabilisers then alter the orientation of the ship.

In a similar fashion airplanes are kept level, virtual reality headsets can detect head movements and the international space station is kept in orbit. Also in recent years ^[5] Dean Kamen of the Segway company unveiled the personal transporter - a simple design which used gyroscopes to provide information to motors which keeps the rider stable.

The Hubble Space Telescope ^[3] is steadily orientated to view the stars using gyroscopes, however whereas the previously mentioned examples used gyroscopes to give measurements, Hubble actually uses three gyroscopes to orientate itself. The reason for this is simply that using rockets emits pollution which hinders the telescope's observations.

Gyroscopic motion must be considered in the design of machines where the components rotate. In particular, a design to save kinetic energy whilst the car is braking was rejected due to a number of reasons.

When a car brakes, the brake pads slow the car by transferring the car's kinetic energy into heat. The rejected design ^[6] involved a mechanism to slow the car by transferring the kinetic energy of the car to a flywheel (a heavy rotating disk). Then when the car is ready to move again, the energy stored in the flywheel can be transferred back to the wheels.

The design was rejected for two main reasons. Firstly the gyroscopic motion of the large flywheel could loosen the fixings and so be dangerous (i.e. as it flies down the street). The

second reason is that when the mass of the flywheel is considered, the gyroscopic motion itself could alter the motion of the entire car.

In the previous example, the mass of the gyroscope had to be considered in relation to the problem. If we consider the mass of a rotating compact disk and the likelihood of the disk not being horizontal. We find that we need not worry about the disk precessing or nutating as such motions would be so small.

The designers of American Indy racing cars realised that, as the car only ever turns in one direction (around the oval track), the rotating engine could provide gyroscopic forces to push the car towards the ground. As a result this increased friction and so further prevented the car from spinning off the track.

It is not just machines where gyroscopic motion must be considered. St. John's tower in Liverpool (which now houses Radio City) is a tower which has a rotating disk at the top. The large mass of this rotating disk meant that precession and nutation had to be considered in the design of the tower. Without these considerations the entire tower could easily have been unstable.

A final example of using gyroscopes was found in physical training. A spinning gyroscope was fixed inside a plastic shell; therefore when the shell is rotated the gyroscope resists the rotation. The idea is that the athlete holds the device in their hand and twists their arm. To accomplish this they must overcome the resistance and so exercise the muscles in their arms.

The Future

Although the Hubble Space Telescope uses gyroscopes to orientate itself to point at different celestial bodies, gyroscopes are not yet used as a means of transport. The motivation for such a propulsion system is that, although it may not be powerful enough to work on Earth, it could be entirely solar powered which would make it ideal for space. This would mean that combustible fuel would not be required, thereby saving cost and mass.

Conclusion

Gyroscopes are interesting because of their many different uses, it is interesting to see how one simple aspect of physics can be manipulated to suit so many varied situations. Gyroscopic motion itself is interesting because the motion of spinning bodies is so unlike the motion of non-spinning bodies. When you consider the how many bodies are rotating in everyday life, you soon realise how important it is to have an understanding of gyroscopes.

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